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**NATURE OF FLUID FLOWS IN DIFFERENTIALLY HEATED  
CYLINDRICAL CONTAINER FILLED WITH A STRATIFIED SOLUTION**

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Semiconductor crystals such as  $Hg_{1-x}Cd_xTe$  grown by unidirectional solidification Bridgmann method have shown compositional segregations in both the axial and radial directions (Lehoczky et. al. 1980, 1981, 1983). Due to the wide separation between the liquidus and the solidus of its pseudobinary phase diagram (Lehoczky and Szofran 1981), there is a diffusion layer of higher HgTe content built up in the melt near the melt-solid interface which gives a solute concentration gradient in the axial direction. Because of the higher thermal conductivity in the melt than that in the crystal there is a thermal leakage through the fused silica crucible wall near the melt-solid interface. This gives a thermal gradient in the radial direction. Hart (1971), Thorpe, Hutt and Soulsby (1969) have shown that under such condition a fluid will become convectively unstable as a result of different diffusivities of temperature and solute. It is quite important to understand the effects of this thermosolute convection on the compositional segregation in the unidirectionally solidified crystals. To reach this goal, we start with a simplified problem. We study the nature of fluid flows of a stratified solution in a cylindrical container with a radial temperature gradient. The cylindrical container wall is considered to be maintained at a higher temperature than that at the center of the solution and the solution in the lower gravitational direction has higher solute concentration which decrease linearly to a lower concentration and then remain constant to the top of the solution (see Fig. 1). The sample solution is taken to be salt water with its properties listed in table 1.

Table 1. Material Properties for 3.5% salt water

Mass diffusivity for salt water	$\alpha_C = 1.3E-9 \text{ m}^2/\text{s}$
Thermal diffusivity	$\alpha_T = 1.5E-7 \text{ m}^2/\text{s}$
Density of salt water	$\rho = 920 \text{ Kg/m}^3$
Viscosity of salt water	$\mu = 7.6E-4 \text{ Kg/m-s}$
Kinematic viscosity	$\nu = 8.26E-7 \text{ m}^2/\text{s}$
Prandtl number	$Pr = \nu/\alpha_T = 5.65$
Schmidt number	$Sc = \nu/\alpha_C = 635.4$
Volume expansion coefficient ( thermal)	$\beta_T = 2.75E-4$
Diameter of the cylinder	$R = 0.005 \text{ m}$
Grashof number for temperature	$Gr_T = 22.23^2$

Under the Boussinesq approximation, the vector form of the governing equations for incompressible fluid flow of the system are:

Momentum Conservation

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p - g(\beta_T(T-T_0) + \beta_C(C-C_0)) \hat{e}_z + \nu \nabla^2 \vec{u} \quad [1]$$

Continuity Equation

$$\nabla \cdot \vec{u} = 0 \quad [2]$$

Energy Conservation

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \alpha_T \nabla^2 T \quad [3]$$

### Solute Conservation

$$\frac{\partial C}{\partial t} + \vec{u} \cdot \nabla C = \alpha_C \nabla^2 C \quad [4]$$

Where  $\vec{u}$  is fluid flow velocity, T and C are the temperature and the solute concentration of the fluid, respectively, and  $\alpha_T$  and  $\alpha_C$  are the diffusivity for temperature and solute, respectively.

For axial symmetric boundary conditions, the governing equations in cylindrical coordinates are:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \omega \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right), \quad [5]$$

$$\begin{aligned} \frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial r} + \omega \frac{\partial \omega}{\partial z} = & -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial z^2} \right) \\ & + g(\beta_T(T - T_0) + \beta_C(C - C_0)), \end{aligned} \quad [6]$$

$$\frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial \omega}{\partial z} = 0, \quad [7]$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + \omega \frac{\partial T}{\partial z} = +\alpha_T \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right), \text{ and} \quad [8]$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial r} + \omega \frac{\partial C}{\partial z} = \alpha_C \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right). \quad [9]$$

The equations can be nondimensionalized by setting the temperature and solute concentration as

$$T = T_0 + \Delta T T^* \quad \text{and} \quad C = C_0 + \Delta C C^*, \text{ and}$$

scaling the variables by a factor F; i. e.  $V = F V^*$ . Scaling length by R, velocity by  $\sqrt{g\beta_T \Delta T R}$ , time by  $R/\sqrt{g\beta_T \Delta T R}$ , pressure by  $\rho g \beta_T \Delta T R$ , temperature by  $\Delta T$  and solute by  $\Delta C$ . After the scaling and dropping all the \*, the dimensionless equations become:

$$Gr_T^{1/2} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \omega \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right), \quad [10]$$

$$\begin{aligned} Gr_T^{1/2} \left( \frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial r} + \omega \frac{\partial \omega}{\partial z} \right) = & -\frac{\partial p}{\partial z} + \left( \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial z^2} \right) \\ & + Gr_T^{1/2} (T + (Gr_C/Gr_T) C), \end{aligned} \quad [11]$$

$$Pr Gr_T^{1/2} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + \omega \frac{\partial T}{\partial z} \right) = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}, \quad \text{and} \quad [12]$$

$$Sc Gr_T^{1/2} \left( \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial r} + \omega \frac{\partial C}{\partial z} \right) = \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}. \quad [13]$$

Where the thermal and solutal Grashof numbers respectively,  $Gr_T$ ,  $Gr_C$  are defined by:

$$Gr_T = \frac{g \beta_T \Delta T R^3}{\nu^2} \quad \text{and} \quad Gr_C = \frac{g \beta_C \Delta C R^3}{\nu^2}$$

The Prandtl number,  $Pr$ , and Schmidt number,  $Sc$ , are defined by:

$$Sc = \nu / \alpha_C \quad \text{and} \quad Pr = \nu / \alpha_T$$

These governing equations show that the flow characteristic are determined uniquely by  $Gr_T$ ,  $Gr_C$ ,  $Pr$  and  $Sc$ . These equations have been solved by the FIDAP program developed by Fluid Dynamics International, Inc. Preliminary results reveal that any convection influences the solute field much more than it does the temperature field. Since the Prandtl and Schmidt numbers are properties of the particular fluid, the variations with  $Gr_T$  and  $Gr_C$  were primarily studied. Figure 1 through 4 depict the computational domain and some of the results obtained.

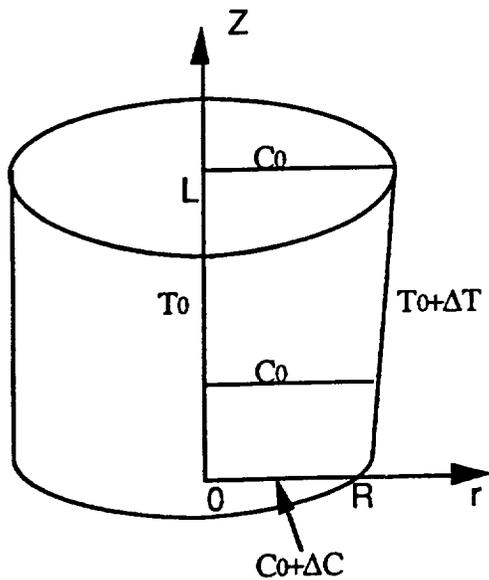


Fig. 1. Problem set up

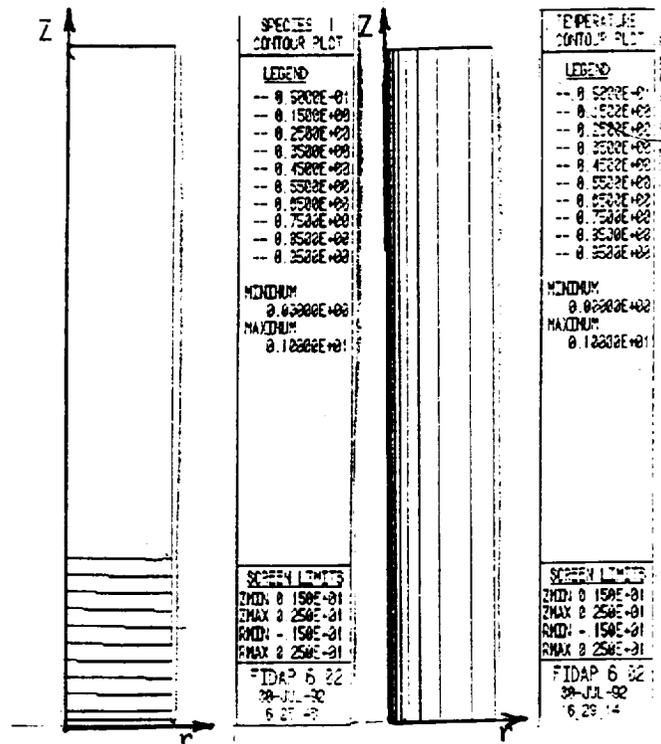


Fig. 2. Concentration and temperature fields for  $Gr_T = 0.04$  and  $Gr_C/Gr_T = 1.0$

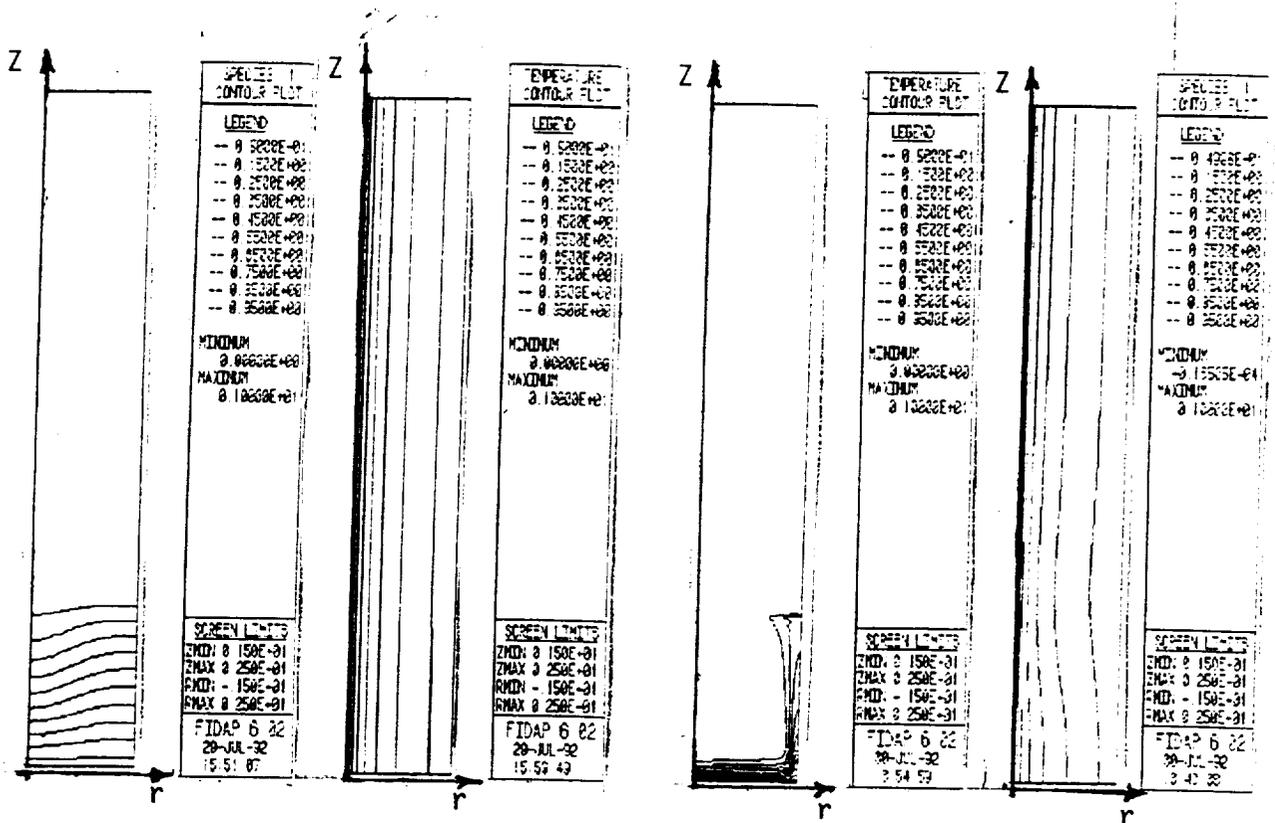


Fig. 3. Concentration and temperature fields for  $Gr_T = 0.49$  and  $Gr_C/Gr_T = 1.0$

Fig. 4. Concentration and temperature fields for  $Gr_T = 0.49$  and  $Gr_C/Gr_T = 1000$

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